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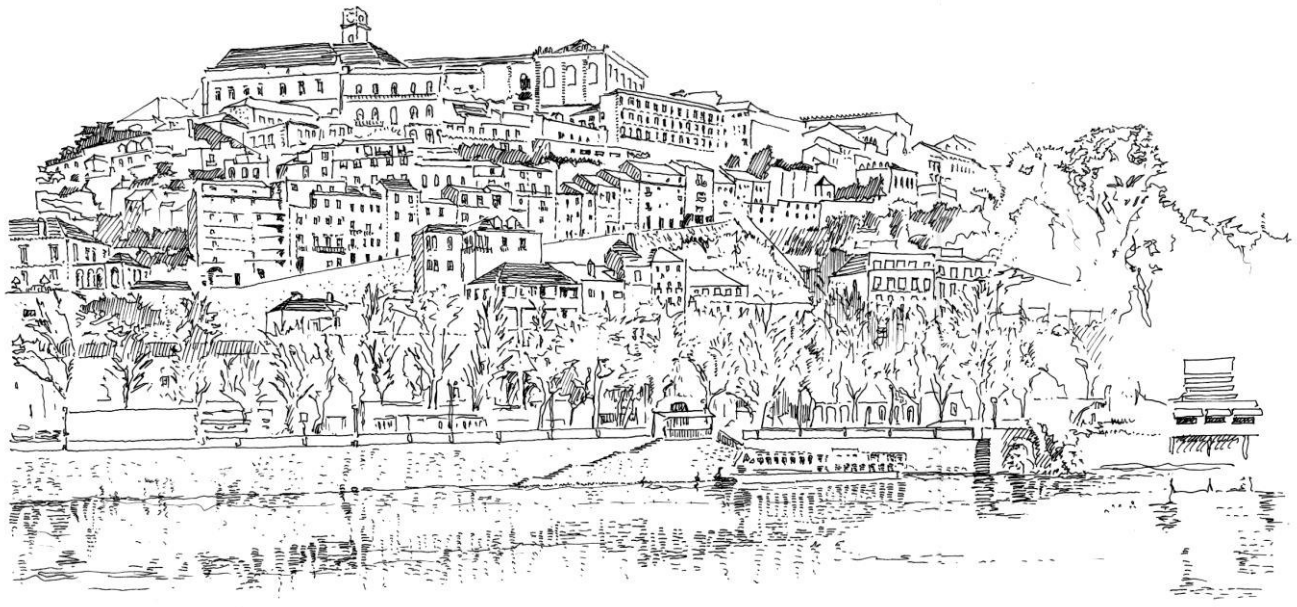

Coimbra
by Rui Barreiros Duarte

Book of abstracts

Contents

The average singular value of a complex random matrix decreases with dimension

Luís Daniel Abreu

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Abstract

We prove that the average singular value of a complex valued random matrix X with random i.i.d., N (0,1) entries, decreases monotonically with d to the limit given by the Marchenko-Pastur distribution. This has recently been conjectured by Afonso S. Bandeira, Cristopher Kennedy and Amit Singer in their study of the Little Grothendieck problem over the unitary group, a combinatorial optimization problem.

Mathematical aspects of Non-Hermitian quantum mechanics

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Abstract

The relevance in Physics of non-Hermitian operators with real eigenvalues is being widely recognized not only in Quantum Mechanics but also in other areas of Quantum Physics such as quantum fluid dynamics, quantum field theory, and so on. In this note, mathematical aspects of a system of two interacting bosons described by a non-Hermitian Hamiltonian are investigated. The real eigenvalues of the Hamiltonian are explicitly determined as well as complete biorthogonal sets of eigenvectors for the Hamiltonian and its adjoint. It is argued that the existence of an involutive operator J which renders the Hamiltonian J-Hermitian leads to the unambiguous definition of an associated positive definite norm allowing for the standard probabilistic interpretation of Quantum Mechanics.

On spectral properties of symmetric and skew-symmetric operators

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Abstract

In this talk, we show many spectral properties that are inherited by m-complex symmetric and m-skew complex symmetric operators and give new results or recapture some known ones for complex symmetric operators.

Keywords: Spectral theory, complex symmetric operator, Bishop's property(β).

A Riemann-Hilbert approach to rotating attractors

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Abstract

Extremal rotating black holes provide examples of dynamical systems that exhibit the attractor mechanism. These gravitational solutions arise as solutions to a set of two-dimensional non-linear field equations that can be viewed as integrability conditions for an auxiliary linear system, called the Breitenlohner-Maison (BM) linear system. Solutions to this linear system can, in turn, be constructed by means of an associated Riemann-Hilbert matrix factorization problem. Using this reformulation, we show how to obtain explicit (novel) rotating attractor solutions as solutions to the aforementioned linear system.

Work in collaboration with Cristina Câmara, Thomas Mohaupt and Suresh Nampuri.

Keywords: rotating attractors, dynamical systems, linear systems, Riemann-Hilbert factorization problem.

Factorization of operators as an inverse problem

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Abstract

We study several examples of factorizations of bounded linear operators on a Hilbert space $\mathcal{H}$, following the next scheme. Let $\mathcal{A}$ and $\mathcal{B}$ be two subsets of $L(\mathcal{H})$:

Problem 1: Characterize the set $\mathcal{AB}$ of all products of an operator in $\mathcal{B}$ by an operator in $\mathcal{A}$.

Problem 2: Given $T \in \mathcal{AB}$, describe the set of all pairs $(A, B) \in \mathcal{A} \times \mathcal{B}$ such that $AB = T$.

Problem 3: Find the best pair (A, B) as before, for some natural optimality condition.

We present several results on these lines, if $\mathcal{A}$ and $\mathcal{B}$ are classical subsets of operators (i. e., partial isometries, orthogonal or oblique projectors, unitaries, positive operators,...).

Keywords: factorization of operators, projections, partial isometries.

Hyponormality of commuting pairs of Toeplitz operators with matrix-valued symbols

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Abstract

In joint work with In Sung Hwang and Woo Young Lee, we consider commuting pairs of Toeplitz operators on the Hardy space of the unit circle, whose symbols are matrix-valued. We obtain a complete characterization

of (joint) hyponormality when the symbols are of bounded type. As in the case of scalar-valued symbols (cf. R.E. Curto and W.Y. Lee, Joint hyponormality of Toeplitz pairs, Memoirs AMS 150, no. 712(2001)), we reduce joint hyponormality to the hyponormality of single Toeplitz operators (with matrix-valued symbols).

As a corollary, we obtain a simple description of (joint) hyponormality when the symbols are rational matrix-valued. Along the way we study the self-commutators of Toeplitz pairs with matrix-valued rational symbols, and derive the associated rank formulae.

Keywords: Toeplitz pairs, matrix functions of bounded type, coprime factorization, jointly hyponormal.

Dilations and Constrained Algebras

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Abstract

It is a well known fact that for the disk, von Neumann's inequality holds; that is, if T is a contraction and f is a function analytic in a neighborhood of the disk, then $\|f(T)\| \leq \|f\|$, where the left hand norm is the operator norm, and the right hand norm is the supremum norm over the disk. A simple proof is obtained by invoking the Sz.-Nagy dilation theorem, which states that a contraction dilates to a unitary operator, and then using the spectral theorem for normal operators. A similar result holds over the bidisk by Ando's thorem, which states that a pair of commuting contractions dilates to a pair of commuting unitaries. Halmos asked if it was generally the case that if von Neumann's inequality holds for a single operator on some domain in the complex plane, is it the case that the operator dilates to a normal operator with spectrum in the boundary of the domain? This question also makes sense in the multivariate setting (with dilation to a tuple of commuting normal operators), and in both cases is referred to as the "rational dilation problem". There is a small (though growing) number of situations in which the problem has been addressed. We look at that of the so-called Neil parabola — the subset of points in the closed bidisk satisfying the equation $z^3 = w^2$. As a subset of the bidisk (where rational dilation holds) one might suspect that it should also hold for the Neil parabola. Then again, ...

This is joint work with Michael Jury and Scott McCullough.

Analytic structure of weighted shifts on directed trees

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Abstract

The class of weighted shifts on directed trees generalizes classical weighted shifts. It was intr oduced a few years ago and provided a lot of examples of operators with interesting properties. In the talk I will concentrate on analytic properties of these operators. In particular, I will prove that a weighted shift on a directed tree is related to a multiplier algebra of coefficients of analytic functions. This result leads to a kind of functional calculus for functions from multiplier algebras. Furthermore, I will present some properties of the spectrum and bounded point evaluations of these operators.

The talk is based on a joint work with P. Budzynski and M. Ptak.

Weighted composition operators on the Dirichlet space: boundedness and spectral properties

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Abstract

We will discuss boundedness of weighted composition operators $W_{u,\varphi}$ acting on the classical Dirichlet space $\mathcal{D}$ as $W_{h,\varphi}f = h\,(f \circ \varphi)$ in terms of the *multiplier space* associated to the symbol φ, i.e., $\mathcal{M}(\varphi) = \{u \in \mathcal{D} : W_{u,\varphi} \text{ is bounded on } \mathcal{D}\}$. As one may guess, a prominent role is played by the multipliers of the Dirichlet space. As a consequence, we show the spectrum of invertible weighted composition operators $W_{u,\varphi}$ in $\mathcal{D}$, extending a recent work of Hyvärinen, Lindström, Nieminen and Saukko [2] to the context of the Dirichlet space and address some open questions at this regards.

(Joint work with I. Chalendar (Lyon) and J. R. Partington (Leeds))

References

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Generalized inverses over rings with Jacobson radical

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Abstract

It is already known from literature that some rings, such as local rings and artinian rings, are of the form PEQ+J, where P, Q are invertible, E is idempotent and J is a matrix over the Jacobson radical. Consequently we are concerned with a sum of a von Neumann regular matrix and a matrix over the Jacobson radical. We establish necessary and sufficient conditions for the existence of a generalized inverse and we show applications to formal power series over normed rings and to Roths matrix equation over the ring of upper triangular matrices.

Keywords: Jacobson radical, von Neumann regularity, Toeplitz matrix.

Conic lifts of convex sets through slack operators

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Abstract

In this talk we will address the basic geometric question of when is a given convex set the image under a linear map of an affine slice of a given closed convex cone. Such a representation or 'lift' of the convex set is especially useful if the cone admits an efficient algorithm for linear optimization over its affine slices. The existence of a lift of a convex set to a cone turns out to be equivalent to the existence of a factorization of an operator associated to the set and its polar via elements in the cone and its dual. This generalizes a theorem of Yannakakis that established a connection between polyhedral lifts of a polytope and nonnegative factorizations of its slack matrix.

Based on joint work with Pablo Parrilo and Rekha Thomas.

Keywords: Convex geometry, Constrained operator factorizations, cone ranks of matrices.

Multipliers between model spaces

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Abstract

Model spaces were probably first introduced in operator theory in connection with the function model of Nagy-Foias, but play now an important role in many areas of modern analysis. In this talk we are interested in multipliers from one model space to another one. Our central result asserts that these can be characterized in terms of suitable kernels of Toeplitz operators and

Carleson measures. This problem has previously been considered by Crofoot in the situation of onto multipliers.

Here, we will first discuss a certain number of general results (e.g. finite dimensional case, the case when one model space is contained in another one, etc.).

Then, we will consider a certain number of more specific situations. For instance, using a sub-level set condition of Cohn, we show that under certain circumstances every multiplier between the two model spaces is a bounded function. However, this is not always the case as we will see in different examples. In particular, in the case of onto multipliers, this answers a question posed by Crofoot.

When considering model spaces of the upper-half plane, we will discuss in some detail when the associated inner function is meromorphic inner. This connects to de Branges spaces of entire functions which are closely related to different important problems in complex analysis (e.g., zero distribution, differential equations, and completeness problems). When the derivative of the associated inner function is bounded, we show that the multipliers can be described as the kernel of a certain Toeplitz operator without appealing to the Carleson measure condition.

(This is joint work with E. Fricain and W.T. Ross.)

Bloch functions and asymptotic tail variance

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Abstract

The boundary growth of Bloch functions has been studied for a long time. However, this has not been studied with care for detail, as needed e.g. in the study of the fractal behavior of quasicircles. We obtain a sharp exponential square integrability theorem, which reminds of theorems of Beurling and of John-Nirenberg. This theorem also gives asymptotically sharp bounds on the integral means spectrum.

Subnormality of the Cauchy Dual of a 2-isometry

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Abstract

Let T be a bounded linear operator on a complex Hilbert space $\mathcal{H}$. W say that T is a 2-isometry, if $I - 2T^*T + T^{*2}T^2 = 0$. The Cauchy dual T' of T is given by $T' = T(T^*T)^{-1}$. In the talk we give an answer to the following question: is the Cauchy dual of a 2-isometry a subnormal contraction? We present several interesting classes of 2-isometries arising from weighted shifts on rooted and leafless directed trees.

Keywords: 2-isometry, Cauchy dual, directed trees.

Matrix Completion Problems: History and the Future

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Abstract

A partial matrix is a rectangular array in which some entries are specified and the other, unspecified, entries are free to be chosen. A completion of a partial matrix is a choice of values for the unspecified entries, resulting in a conventional matrix. A matrix completion problem asks which partial matrices have a completion with a given desired property. Often, the pattern of specified entries, described by a graph, plays a significant role in understanding a completion problem. We describe some historical and recent work on the positive definite and totally positive completion problems (and others), some general theory, some recent work and some interesting problems that might help to deepen what is known.

One-sided invertibility criteria for binomial functional operators with shift and slowly oscillating data

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Abstract

Let α be an orientation-preserving homeomorphism of $[0,\infty]$ onto itself with only two fixed points at 0 and ∞, whose restriction to $\mathbf{R}_+ = (0,\infty)$ is a diffeomorphism, and let U_α be the corresponding isometric shift operator acting on the Lebesgue space $L^p(\mathbf{R}_+)$ by the rule $U_\alpha f = (\alpha')^{1/p}(f \circ \alpha)$. We prove criteria for the one-sided invertibility of the binomial functional operator $aI - bU_\alpha$ on the spaces $L^p(\mathbf{R}_+)$, $p \in (1,\infty)$, under the assumptions that a, b and α' are bounded and continuous on $\mathbf{R}_+$ and may have slowly oscillating discontinuities at 0 and ∞.

Keywords: Orientation-preserving non-Carleman shift, slowly oscillating function, limit operator, one-sided invertibility.

Cyclicity in the harmonic Dirichlet space

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Abstract

The harmonic Dirichlet space is the Hilbert space of functions $f \in L^2(T)$ such that

$$\|f\|^2_{\mathcal{D}(T)} := \sum_{n=-\infty}^{+\infty} (1+|n|)|\hat{f}(n)|^2 < \infty.$$

We give sufficient conditions for f to be cyclic in $\mathcal{D}(T)$, in other words, for $\{\zeta^n f(\zeta) : \ n \geq 0\}$ to span a dense subspace of $\mathcal{D}(T)$. Joint work with Evgueni Abakumov, Omar El-Fallah and Thomas Ransford.

Keywords: Harmonic Dirichlet space, capacity, cyclic vectors.

Uniqueness of the numerical range of truncated shifts

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Abstract

The simple unilateral shift S can be represented as multiplication by the identical function $\chi(z) = z$ on the Hardy–Hilbert space H^2. The function $\vartheta \in H^\infty$ is inner, if $|\vartheta(\zeta)| = 1$ holds for almost every ζ on the unit circle $\mathbb{T}$. The model space, associated to ϑ, is $\mathcal{H}(\vartheta) = H^2 \ominus \vartheta H^2$. The truncated shift $S(\vartheta)$, defined as the compression of S to $\mathcal{H}(\vartheta)$: $S(\vartheta) = P_{\mathcal{H}(\vartheta)} S | \mathcal{H}(\vartheta) \in \mathcal{L}(\mathcal{H}(\vartheta))$, is a cyclic C_0-contraction; its detailed study can be found, e.g., in the books by H. Bercovici and N. K. Nikolski. The Hilbert space $\mathcal{H}(\vartheta)$ is finite dimensional exactly when ϑ is a finite Blaschke product. In that case H.-L. Gau and P. Y. Wu showed (1998) that the numerical range $W(S(\vartheta))$ uniquely determines ϑ. This uniqueness statement was extended by I. Chalendar, P. Gorkin and J. R. Partington (2011) to the case when $W(S(\vartheta))^- \cap \mathbb{T} = \{\lambda_0\}$ is a singleton. We investigate the case, when $W(S(\vartheta))^- \cap \mathbb{T}$ consists of two points; in particular, when $W(S(\vartheta))^- \cap \mathbb{T} = \{-1, 1\}$.

It is known that $W(S(\vartheta))^- = \cap_\lambda W(U_\lambda)^-$, where U_λ $(\lambda \in \mathbb{T})$ are the unitary dilations of $S(\vartheta)$ on the space $\mathcal{H}(\check{\vartheta})$ with $\check{\vartheta} = \chi\vartheta$. Furthermore, the point spectrum of the diagonal operator U_λ coincides with the level set $L(\check{\vartheta}, \lambda) = \{\zeta \in \mathbb{T} : \check{\vartheta}(\zeta) = \lambda\}$. Applying fractional linear mappings, the problem can be transferred to uniqueness questions in the Nevanlinna class $\mathcal{N}$ of analytic selfmappings of the upper halfplane $\mathbb{C}_+$. H. Bercovici and D. Timotin (2012) made a thorough study of $\mathcal{N}$ from this perspective. It turned out that coincidence of two level sets: $L(\check{\vartheta}, \lambda_1) = L(\check{\eta}, \mu_1)$, $L(\check{\vartheta}, \lambda_2) = L(\check{\eta}, \mu_2)$, already implies coincidence of the inner functions: $\eta = \kappa\vartheta$ $(\kappa \in \mathbb{T})$. The basic problem remains whether $W(S(\vartheta))^- = W(S(\eta))^-$ ensures coincidence of two level sets or not. A related question is whether the frequency function of $\check{\vartheta}$ on $\mathbb{T}_+ = \mathbb{T} \cap \mathbb{C}_+$, which is connected with the shape of $W(S(\vartheta))^- \cap \mathbb{C}_+$, determines $\check{\vartheta}$. We present partial results, reducing the problem to a particular case.

On characterizations of asymmetric truncated Toeplitz operators

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Abstract

Consider two nonconstant inner functions α and θ such that α divides θ. For a function $\varphi \in L^2$ we can define an asymmetric truncated Toeplitz operator

$$A_\varphi \colon H^2 \ominus \theta H^2 \to H^2 \ominus \alpha H^2$$

by the formula $A_\varphi f = P_\alpha(\varphi f)$, where $P_\alpha \colon L^2 \to H^2 \ominus \alpha H^2$ is the orthogonal projection. During the talk we will investigate some properties of bounded asymmetric truncated Toeplitz operators with L^2 symbols. In particular, we will give two characterizations of such operators in terms of specific operators of rank two.

Keywords: asymmetric truncated Toeplitz operator, model space.

What is the optimal geometry for the lowest eigenvalue of the Laplace operator?

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Abstract

We give an expository talk on the optimisation of the lowest eigenvalue of the Laplace operator with respect to the geometry of a domain with fixed volume. It is well known that the ball is always the minimiser for this kind of spectral isoperimetric problems under Dirichlet, Neumann or positive Robin boundary conditions. We give a counterexample to the long standing conjecture that the ball is an optimiser as well if the Robin boundary conditions are negative. On the favourable side, we show that the conjecture holds in two dimensions provided that the boundary parameter is small.

This is joint work with Pedro Freitas published in Advances in Mathematics (2015).

An extension of a recent determinantal inequality by Audenaert arising in MRI

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Abstract

Recently, K. Audenaert [1] obtained a determinantal inequality, which has arisen in the study of interpolation methods for image processing in diffusion tensor imaging, used in medical MRI (magnetic resonance imaging), from

a log-majorisation result due to Matharu and Aujla [2]. Majorization is a powerful tool for establishing matrix inequalities and some classical determinantal inequalities can find their majorization counterparts. We generalize Matharu and Aujla's log-majorization and present its relation with other well-known operator inequalities, such as Furuta inequality. In particular, a reformulation of the following determinantal inequality

$$\det(A + U^*B) \leq \det(A + B),$$

formulated by Audenaert, when comparing geodesics induced by different metrics, for positive semi-definite matrices A and B, where U is a unitary matrix that appears at the polar decomposition of BA, is extended.

Keywords: majorization, determinantal inequalities.

References

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Weighted Procrustes problems

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Abstract

Let $\mathcal{H}$ be a Hilbert space, $L(\mathcal{H})$ the algebra of bounded linear operators on $\mathcal{H}$ and $W \in L(\mathcal{H})$ a positive operator such that $W^{1/2}$ is in the p-Schatten class, for some $1 \leq p < \infty$. Given $A \in L(\mathcal{H})$ with closed range and $B \in L(\mathcal{H})$, we study the following weighted approximation problem: analize the existence of

$$\min_{X \in L(\mathcal{H})} \|AX - B\|_{p,W}$$

where $\|X\|_{p,W} = \|W^{1/2}X\|_p$.

We prove that the existence of this minimum is equivalent to a compatibility condition between $R(B)$ and $R(A)$ involving the weight W and we characterize the operators which minimize this problem as W-inverses of A in $R(B)$.

Keywords: Schatten norm, Procrustes problems, Compatibility.

Constructive approximation in $H(b)$ spaces

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Abstract

In several Banach spaces of analytic functions on the open unit disc D, polynomials are dense. The standard technique is to approximate a function f by its dilation f_r. The latter is a function which is defined on a disc larger than the open unit disc and thus one expects a better behavior for f_r on D. For example, we should be able to approximate f_r by polynomials on D. Therefore, if r is close enough to one, these polynomials are also good approximations for f. This general approach works in many function spaces, e.g Hardy , Bergman and Dirichlet spaces. However, and amazingly, it dramatically fails in de Branges-Rovnyak spaces and at the same time polynomials are still dense in these spaces. The first proof was non-constructive. Using Toeplitz operators, we provide a new constructive proof.

Joint work with O. El-Fallah, E. Fricain, K. Kellay, T. Ransford

Homomorphism to $\mathbb{R}$ obtained from a quasimorphism with an application to the reduced group C^*-algebra

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Abstract

Let G be a finitely generated, discrete group. Using a random walk approach, Erschler and Karlsson constructed a homomorphism $G \to \mathbb{R}$. Central to their construction were the word length ℓ and a well behaved measure μ on G. We consider a modified version of this construction using instead of ℓ a quasimorphism f of G. Moreover, if a group H acts on G via group

automorphisms we show that this technique can be adapted to construct a homomorphism of the semidirect product $G \rtimes_\phi H$ to $\mathbb{R}$.

As an application, we construct a $*$-automorphism α_f of the underlying reduced group C^*-algebra $C_r^*(G)$.

Keywords: Random walks on groups, quasimorphisms, $*$-automorphism.

Systems of Dilated Functions: completeness, minimality, basisness

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Abstract

We analyze the system

$$u_n(x) = S(nx), \quad 0 \le x \le \pi, \quad n \in \mathbb{N},$$

where $S(x)$ is a trigonometric polynomial

$$S(x) = \sum_{\alpha \in A} s(\alpha) \sin\left(k(\alpha)x\right), \quad \#A < \infty, \quad A \in \mathbb{N}_0^m,$$
$$k(\alpha) = \prod_{j=1}^{m} p_j^{\alpha_j}, \quad \alpha = (\alpha_j)_{j=1}^m,$$

where the p_j are prime numbers, $1 \le j \le m$. In terms of the algebraic polynomial on $\mathbb{C}^m$,

$$a(w) = \sum_{j=1}^{m} s(\alpha) w^\alpha, \quad w^\alpha = \prod_{j=1}^{m} w_j^{\alpha_j},$$

and its zero set $Z(a)$, many questions on completeness, minimality, and basisness of the system $\{u_n\}$ can be answered. (To some extent, we follow [1], [2], [3], [4].)

One of our claims: define $\overline{\mathcal{D}}^m$ to be the closed unit m-disc in $\mathbb{C}^m$, and $\mathbb{T}^m$ to be the Shilov boundary of $\overline{\mathcal{D}}^m$. If the intersection $Z(a) \cap \overline{\mathcal{D}}^m = Z(a) \cap \mathbb{T}^m \neq \emptyset$ then the system $\{u_n\}$ is complete but not a basis in $L^p[0,\pi]$, $1 < p < \infty$.

For $m \ge 4$, a few elementary examples demonstrate the need for the Muckenhoupt criterion in the multidimensional case.

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Intervals of density of an exponential polynomial

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Abstract

Given an exponential polynomial $h(z) := 1 + \sum_{k=1}^{N} a_k e^{-zr_k}, \quad z = x + iy$, under certain conditions imposed to the vectors $a = (a_k)_{k=1,\dots,N}$ and $r = (r_k)_{k=1,\dots,N}$, the set $R_{h(z)} := \overline{\{\Re z : h(z) = 0\}}$ can contain non-degenerate intervals called *intervals of density*. The aim of this talk is to study what conditions on a and r are needed to assure the existence of intervals of density of $h(z)$ having maximal length.

Approximation numbers estimates via continuity envelopes

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Abstract

In this talk we present the general concept of continuity envelope of a function space, introduced by D.D. Haroske and H. Triebel. The case of smoothness Morrey spaces is considered and, as a consequence, we obtain the asymptotic behaviour of the approximation numbers of some embeddings of smoothness Morrey spaces into the space of bounded uniformly continuous functions in the unit ball.

The talk is based on a joint work with W. Yuan, D.D. Haroske, L. Skrzypczak and D. Yang.

Keywords: continuity envelope, smoothness Morrey spaces, approximation numbers.

Circles in the spectrum and the geometry of orbits

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Abstract

We prove that a bounded linear Hilbert space operator has the unit circle in its essential approximate point spectrum if and only if it admits an orbit satisfying certain orthogonality and almost-orthogonality relations. As consequences, we derive in particular wide generalizations of Arveson's theorem as well as show that the weak convergence of operator powers implies the uniform convergence of their compressions on an infinite-dimensional subspace.

Lie modules and Lie ideals of operator algebras

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Abstract

In this talk we consider the Lie structure of operator algebras having totally ordered subspace lattices. Although a characterisation of the weakly closed bimodules of these algebras has long been established, their Lie structure has proved to be somewhat elusive. In the 90's Hudson, Marcoux and Sourour proved a decisive result towards the characterisation of the weakly closed Lie ideals by showing that these ideals can be encapsulated in appropriate associative ideals and von Neumann algebras. It is the purpose of this talk to show that a similar result holds when passing from Lie ideals to Lie modules.

Keywords: bimodule, Lie ideal, Lie module.

References

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[2] L. Oliveira and M. Santos, Weakly closed Lie modules of nest algebras, *Operators and Matrices*, to appear.

Semigroups acting on Hardy spaces

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Abstract

This talk is based on the preprints [2] and [3], as well as earlier work. It discusses semigroups of operators on spaces such as the Hardy and Dirichlet spaces, for which the generators are differential operators. For example, it is well-known that semigroups of composition operators have generators of the form $f \mapsto Gf'$, where G is holomorphic in the unit disc (see [1] and [4]).

Conditions are given for higher-order differential operators to generate semigroups. Issues such as compactness and analyticity of such semigroups are also discussed.

Keywords: C_0 semigroup, analytic semigroup, compact semigroup, semiflow, Hardy space, composition operator.

References

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[3] I. Chalendar and J. R. Partington, Compactness, differentiability and similarity to an isometry of composition semigroups. Preprint, 2016.

[4] A. G. Siskakis, Semigroups of composition operators on spaces of analytic functions, a review. *Studies on composition operators (Laramie, WY, 1996)*, 229–252, Contemp. Math., 213, Amer. Math. Soc., Providence, RI, 1998.

Quasinormal and composition operators in L^2-spaces

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Abstract

In 1953 A. Brown introduced the class of bounded quasinormal operators. In the case of unbounded operators, two different definitions of unbounded quasinormal operators appeared independently. The first one was given in 1983 by Kaufman, and a few years later, the second one by Stochel and Szafraniec.

The talk will be devoted to the new characterisation of quasinormality and normality of unbounded operators.

Keywords: composition operators in L^2-space, weighted shift on directed tree, quasinormal operators.

Unbounded composition operators via inductive limits: cosubnormals with matrical symbols

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Abstract

The aim of this talk is to present recent results concerning cosubnormality of unbounded composition operators induced by finite and infinite matrix symbols. There is no effective general criterion for subnormality of unbounded operators. As a consequence, the methods of verifying the subnormality of an operator depends on its properties. In the context of the aforementioned class of operators, inductive limit method can be applied. In finite-dimensional case this method enables us to give a criterion for cosubnormality of unbounded composition operators induced by finite invertible matrix symbol.

If the matrix is infinite, then the question of cosubnormality of corresponding composition operator is far more complicated. We introduce classes $\mathcal{S}^*_{n,r}$ of unbounded operators closely related to cosubnormal operators and investigate under what conditions composition operators with infinite matrix symbols belong to the classes. Using inductive limits we go from finite-dimensional case to infinite-dimensional case and get three criteria for cosubnormality.

The talk is based on the joint work with P. Budzyński and P. Dymek.

References

[1] P. Budzyński, P. Dymek, A. Płaneta, Unbounded composition operators via inductive limits: cosubnormal operators with matrix symbols, *Filomat*, (to appear), http://arxiv.org/abs/1502.01638.

[2] P. Budzyński, P. Dymek, A. Płaneta, Unbounded composition operators via inductive limits: Cosubnormal operators with matrix symbols, II BJMA (accepted)

C-symmetry of asymmetric truncated Toeplitz operators

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Abstract

Let H^2 be the Hardy space on the unit disc, identified as usual with a subspace of L^2 on the unit circle. With any nonconstant inner function θ we associate the model space K^2_θ, defined by $K^2_\theta = H^2 \ominus \theta H^2$. In this space we can define the conjugation (antilinear, isometric, involution) $C_\theta \colon K^2_\theta \to K^2_\theta$ by $C_\theta f(z) = \theta \overline{z f(z)}$.

Let us consider two nonconstant inner functions α and θ such that α divides θ. For a given function $\varphi \in L^2$ we can define an asymmetric truncated Toeplitz operator $A_\varphi \colon K^2_\theta \to K^2_\alpha$ by $A_\varphi f = P_\alpha(\varphi f)$, where $P_\alpha \colon L^2 \to K^2_\alpha$ is the orthogonal projection. The relation between bounded asymmetric truncated Toeplitz operators with L^2 symbols and conjugations C_θ, C_α will be investigated. The relations are different than in symmetric case $\theta = \alpha$.

Keywords: asymmetric truncated Toeplitz operator, conjugations.

Matrices over rings of complex functions which are not PIDs

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Abstract

This talk will be concerned with properties of invariant factors of matrices over elementary divisor domains. This class of rings strictly contains principal ideal domains. Certain rings of complex functions give the main examples of EDDs which are not PIDs.

Keywords: invariant factors, elementary divisor domains.

Constant resolvent norm

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Abstract

We discuss the question whether a linear operator can have constant resolvent norm on an open set. We extend the class of operators for which it is known that the latter cannot happen by showing that if the resolvent norm is constant on an open set, then this constant is the global minimum. Several examples, links to the definition of pseudospectra and their convergence will be mentioned as well.

Keywords: resolvent level sets, pseudospectrum.

References

[1] S. Bögli and P. Siegl (2014) Remarks on the Convergence of Pseudospectra, *Integral Equations and Operator Theory* **80**, 303–321.

Normal Matrices subordinate to a tree and flat portions of the field of values

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Abstract

Matrices subordinate to trees are considered. An efficient normality characterization for any such matrix is given, and several consequences (not valid for general normal matrices) of it are established. In addition, the existence (and enumeration) of flat portions on the boundary of the field of values of matrices subordinate to a tree is characterized.

This is joint work with our students Mirjana Stevanovic (NYUAD) and Morrison Turnansky (William and Mary).

Keywords: Field of values, lat boundary portion, normal matrix, tree, undirected graph of a matrix.

Spectral analysis of non-self-adjoint Jacobi operator associated with Jacobian elliptic functions

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Abstract

We provide a detailed spectral analysis of an unbounded Jacobi operator whose diagonal vanishes and the weight sequence is periodically modulated by a complex

coupling parameter. Depending on whether the parameter is on the unit circle or not, the spectrum is either purely discrete or purely essential filling the whole complex plane. The spectral analysis of this operator heavily relies on some new properties of Jacobian elliptic function with complex modulus which have been proved recently in [1]. If time remains these results will be also discussed. This contribution talk is entirely based on results from [1, 2].

Keywords: non-self-adjoint Jacobi operator, Jacobian elliptic functions

References

[1] P. Siegl, F. Štampach (2016) On extremal properties of Jacobian elliptic functions with complex modulus, *Journal of Mathematical Analysis and Applications*, in press, arXiv:1512.06089.

[2] P. Siegl, F. Štampach (2016) Spectral analysis of non-self-adjoint Jacobi operator associated with Jacobian elliptic functions, submitted, arXiv:1603.01052.

Composition operators on Hilbert spaces of entire functions

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Abstract

Composition operators with analytic symbols on reproducing kernel Hilbert spaces over complex Hilbert spaces induced by entire functions with non-negative Taylor's coefficients are discussed. The questions of boundedness, the spectral radius, hyponormality and cohyponormality and so on, are addressed. The case of composition operators on the Segal-Bargmann space of finite and infinite order is discussed with a special care.

Old-times Operator Theory for pretty recent Quantum Mechanics. An example

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Abstract

Based on the paper Katarzyna Górska, Andrzej Horzela and FHSz. Squeezing of arbitrary order: the ups and downs. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* 470 (2014), no. 2172, 20140205, 21 pp.

Commutators, Factorization, BMO and the Hardy Space

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Abstract

In this talk we will revisit an idea of Uchiyama about factorization in Hardy spaces and show how this idea can be implemented in other function spaces. As a result we will obtain factorization theorems for Hardy spaces in multi-parameter settings, multilinear settings, and in the setting of the Bessel operator. Equivalently, we will obtain results about the boundedness of commutators in these settings.

This talk is based on joint work with Ji LI, Xuan Duong, and Donyong Yang.

Keywords: Commutators, Hardy Spaces, BMO.

Joint Spectrum and Infinite Dihedral Group

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Abstract

For a tuple $A = (A_1,\ A_2,\ ...,\ A_n)$ of elements in a unital Banach algebra $\mathcal{B}$, its *projective joint spectrum* $P(A)$ is the collection of $z \in \mathbf{C}^n$ such that the multiparameter pencil $A(z) = z_1A_1 + z_2A_2 + \cdots + z_nA_n$ is not invertible. If $\mathcal{B}$ is the group C^*-algebra for a discrete group G generated by $A_1,\ A_2,\ ...,\ A_n$ with respect to a representation ρ, then $P(A)$ is an invariant of (weak) equivalence for ρ. This talk presents some recent work on the joint spectrum $P(R)$ of $R = (1,\ a,\ t)$ for the infinite dihedral group $D_\infty =< a,\ t \mid a^2 = t^2 = 1 >$ with respect to the left regular representation. Results include a description of the joint spectrum, a formula for the Fuglede-Kadison determinant of the pencil $R(z) = 1 + z_1a + z_2t$, the first singular homology group of the joint resolvent set $P^c(R)$, and dynamiccal properties of the joint spectrum. These results give new insight into some earlier studies on groups of intermediate growth.

Keywords: projective joint spectrum, dihedral group, C^*-algebra, weak containment, Maurer-Cartan form, Fuglede-Kadison determinant.

Posters

Using WOLFRAM MATHEMATICA in spectral theory

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Abstract

Spectral theory has many applications in several main scientific research areas (Structural Mechanics, Aeronautics, Quantum Mechanics, Ecology, Probability Theory, Electrical Engineering, among others) and the importance of its study is globally acknowledged. In recent years, several software applications were made available to the general public with extensive capabilities of symbolic computation. These applications, known as computer algebra systems (CAS), allow to delegate to a computer all, or a significant part, of the symbolic calculations present in many mathematical algorithms. In our work we use the CAS Mathematica to implement for the first time on a computer analytical algorithms developed by us and others within the Operator Theory. The main goal of this work is to present the spectral algorithms [ASpec-Hankel], [ASpecPaired-Scalar], and [ASpecPaired-Matrix] which use the symbolic and numeric computation capabilities of Mathematica to explore the spectra of some classes of singular integral operators, defined on the unit circle. These analytical algorithms allow us to check, for each considered paired singular integral operator, if a complex number (chosen arbitrarily) belongs to its spectrum.

References

[1] Ana C. Conceição and José C. Pereira (2016) Exploring the spectra of some classes of singular integral operators with symbolic computation. *Math.Comput.Sci.*, 10(2). Springer. DOI 10.1007/s11786-016-0264-2. ISSN: 1661-8270.

[2] Ana C. Conceição, Viktor G. Kravchenko, and José C. Pereira (2014) Computing some classes of Cauchy type singular integrals with Mathematica software. *Adv.Comput.Math.* 39(2), 273-288. Springer. DOI 10.1007/s10444-012-9279-7. ISSN 1019-7168.

A singular integral operator with a non-Carleman shift

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Abstract

Some estimates for the dimension of the kernel of the singular integral operator with a non-Carleman shift $I - cUP_+$: $L_2^n(\mathbb{T}) \to L_2^n(\mathbb{T})$, are obtained, where P_+ is the Cauchy projector, U is an isometric shift operator and $c(t)$ is a continuous matrix function.

An extension and a non-unicellularity result in the context of a theorem of Domar

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Abstract

In the setting of weighted L^2-spaces, a striking result due to Domar states that the lattice of closed invariant subspaces for $\{S_\tau\}_{\tau\geq 0}$ in $L^2(\mathbb{R}_+, w(t)dt)$ coincides with the lattice of *"standard invariant subspaces"*

$$L^2([a,\infty), w(t)dt) = \{f \in L^2(\mathbb{R}_+, w(t)dt) : f(t) = 0 \text{ a.e } 0 \leq t \leq a\}, (a \geq 0),$$

whenever w w is positive continuous decreasing function in $\mathbb{R}_+$ such that:

1. $\log w$ is concave in $[c,\infty)$, for some $c \geq 0$.
2. $\lim\limits_{t\to\infty} \dfrac{-\log w(t)}{t} = \infty$ and $\lim\limits_{t\to\infty} \dfrac{\log|\log w(t)| - \log t}{\sqrt{\log t}} = \infty$.

We present an extension of Domar's theorem to a wider class of weight functions w not fulfilling condition (1). In addition, we provide a non-empty class of weights $\tilde{w}$ for which the lattice of closed invariant subspaces for $\{S_\tau\}_{\tau\geq 0}$ in $L^2(\mathbb{R}_+, \tilde{w}(t)dt)$ is non-standard.

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